HW #1

Section 2.1

18. (a) AB is not defined because A is 2×5 and B is 2×2.

(b) $$BA = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 & -2 & 4 \\ 6 & 13 & 8 & -17 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+6(6) & 1(0)+6(13) & 1(3)+6(8) & 1(-2)+6(-17) & 1(4)+6(20) \\ 4(1)+2(6) & 4(0)+2(13) & 4(3)+2(8) & 4(-2)+2(-17) & 4(4)+2(20) \end{bmatrix}$$

$$= \begin{bmatrix} 37 & 78 & 51 & -104 & 124 \\ 16 & 26 & 28 & -42 & 56 \end{bmatrix}$$

38. Expanding the left side of the equation produces

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2a_{11} - a_{21} & 2a_{12} - a_{22} \\ 3a_{11} - 2a_{21} & 3a_{12} - 2a_{22} \end{bmatrix}$$

Since this equals $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. We obtain the system

$$\begin{aligned} 2a_{11} - a_{21} &= 1 \\ 2a_{12} - a_{22} &= 0 \\ 3a_{11} - 2a_{21} &= 0 \\ 3a_{12} - 2a_{22} &= 1. \end{aligned}$$

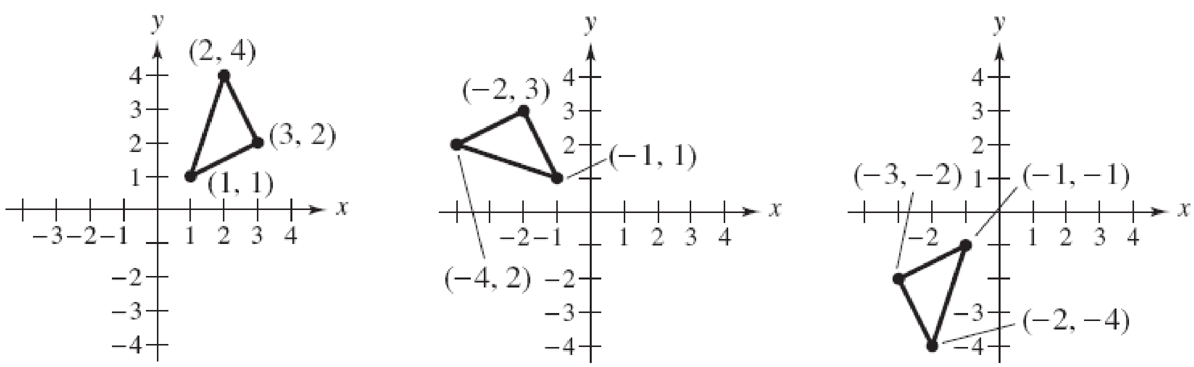
Solving by Gauss-Jordan elimination yields: $a_{11} = 2$, $a_{12} = -1$, $a_{21} = 3$, and $a_{22} = -2$.

So, you have $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$.

62. (a) $$AT = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$AAT = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -3 \\ -1 & -4 & -2 \end{bmatrix}$$

Triangle associated with T | Triangle associated with AT | Triangle associated with AAT



The transformation matrix A rotates the triangle about the origin $90°$ counterclockwise.

2.1 Extra credit:

56. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$. Then $AB - BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is equivalent to

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} - \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Which becomes

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} - (a_{11}b_{11} + a_{21}b_{12}) & don't\ need \\ don't\ need & a_{21}b_{12} + a_{22}b_{22} - (a_{12}b_{21} + a_{22}b_{22}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Cancelling like terms gives

$$\begin{bmatrix} a_{12}b_{21} - a_{21}b_{12} & don't\ need \\ don't\ need & a_{21}b_{12} - a_{12}b_{21} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

And this implies

$a_{12}b_{21} - a_{21}b_{12} = 1$

$a_{21}b_{12} - a_{12}b_{21} = 1$

Since one left hand side is the negative of the other we know that they cannot both be “1”.
(You would get similar terms using the other diagonal. Unfortunately a number and its negative can both equal zero so it does not lead to a contradiction.)

<u>Section 2.2</u>

32. $(AB)^T = \left(\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 1 \\ -4 & -2 \end{bmatrix}^T = \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix}$

36. (a) False. In general, for $n \times n$ matrices A and B it is *not* true that $AB = BA$. For example,

let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$. Then $AB = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = BA$.

(b) True. For any matrix A you have an additive inverse, namely $-A = (-1)A$.
See Theorem 2.2(2) on page 621.

(c) False. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$.

Then $AB = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = AC$, but $B \neq C$.

(d) True. See Theorem 2.6(2) on page 68.

40. $A^{20} = \begin{bmatrix} (1)^{20} & 0 & 0 \\ 0 & (-1)^{20} & 0 \\ 0 & 0 & (1)^{20} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$